Diffraction of Light at Slit and Hindrance

Helmut Nieke

Abstract
As introduction is described that the diffraction figure of the halfplane is building up~ and first in a distance of 10^5 λ appears (by parallel incident light) the accustomed figure. Slit and hindrance of same dimensions have as images in a schlieren apparatus by Abbe on both sides of every edge-image a double-stripe (breadth <0.1 mm) with a small dark space. That means that bent light seems to come out of analogous spheres. Without schlieren apparatus originate edge symmetrical exchanged (equal only in special cases) equal diffraction figures with inner and outer diffraction fringes. General remarks about inner and outer diffraction fringes complete this paper.

I. Diffraction at the half-plane in short distances

Fresnel [1] found experimentally the formula for dependence of distance to the intervals in diffraction-figures at half-planes with divergent to parallel incident light

$$x = w[\lambda s'(1 + s'/s)/2]^{1/2} = w[\lambda s'(s + s')/2 s]^{1/2} \tag{1}$$

where x is the interval of a fringe from the shadow limit, w a running term e.g. 1.21 for the first maximum etc. which are easily to take of Cornu’s spiral, s is the distance from point- or line-formed light-source to half-plane, and s' the distance of half-plane to incident-plane- At very large distances s of light-source or with parallel light formula (1) gets to

$$x = w[\lambda s'/2]^{1/2}. \tag{2}$$

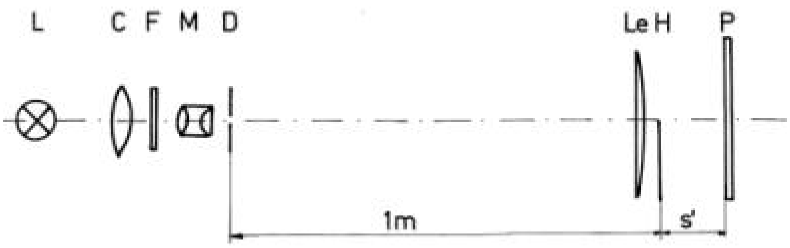

Figure 1. Experimental arrangement for taking photos of the diffraction-figures of half-plane in short distances with parallel incident light. L - light-source, a super-pressure mercury-lamp HBO 100; C - condenser; F - green-filter: M - microscope-objective; D - pinhole-diaphragm, a spinning nozzle 50 mm; Le - lens f' = 1 m; s = 1 m; s' - distance H - P; H - half-plane; P - photofilm.

Here is considered only the light-side diffraction-figure, as known appeared shadow-side a continual falling-off without of periodic. In his measurements Fresnel [1] used distances from 10 cm to some meters. It is the aim of this section to complete this measurements to shorter distances.

The experimental arrangement is shown in fig. 1. To find the spacing from geometric shadow-limit a hindrance is used as by Fresnel, here the edges of a razor-blade. This technical trick is not drawn in fig. 1.

Fig.. 2 shows the experimental results. In a distance of some millimetres the habitual diffraction-figure of half-plane does not appear but only a bright fringe light-side of shadow-limit. First in a distance of 10 mm gradually more and more maxima are forming which become increasingly the spacings as Fresnel described. The first maximum has at first a higher intensity and only at distances of about 64 mm diffraction-figures are found as we know at half-planes. The diffraction figure of half-plane is building up in short distances, then it fits itself to the dependence of distance to equation (2) because the incident light was parallel. Therefore equation (1) and (2) are not valid for very short distances.

At very- great distances (greater than about 10 m) the distance dependence of diffraction fringes seems to approach the proportion linear to the distance, but experiments in these large distances are very difficult.

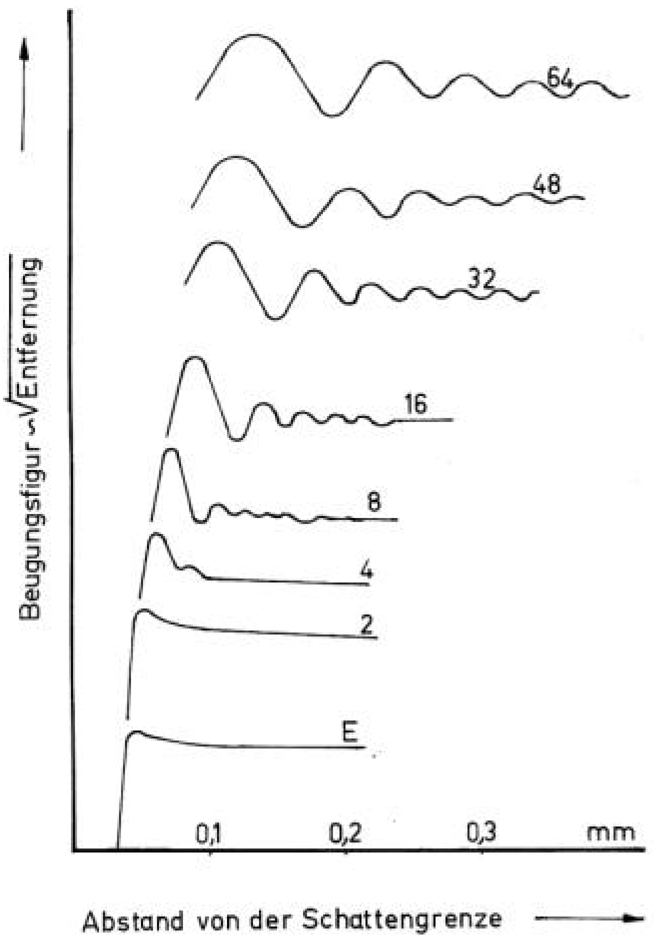

Figure 2. Photometer-curves of diffraction-figures of half-plane in dependence of the distance s' according to figure 1. In the ordinate the photometer-curves are so displaced that every curve comes about to the place that corresponds with the root of the distance s¢ . The marked numbers mean the distance s' in mm; E - film close behind the edge.

For diffraction with electrons first Boersch [2] showed that also a half-plane yields diffraction known by light. Hiller a. Ramberg [3] placed the focus-plane of an electron microscope with an illumination aperture of some 10^{-5} rad in front of and behind the half-plane. With a defocusing of some µm only one high maximum appeared and only at increased defocusing of about 10 µm more maxima are built up. Only at large defocusing of some 10 µm the complete diffraction-figure of the half-plane was found With it a complete analogous results are presented for diffraction at the half-plane in short distances with electrons and in this paper with light if the distances are comparated in so called wave-lengths as unit.

All formulas derived by wave-theory, if noticed or not, are valid for distances s' from the half-plane

$$s' \gg \lambda \qquad (3)$$

This formula can be written more precisely according to these experiments for formula (2) as

$$s' > 10^5\,\lambda. \qquad (4)$$

The distance-dependence (1) or (2) is applied in the formula for the diffraction-figure of half-plane by Fresnel and does not come out of wave-theory.

II. Diffraction at slit and hindrance in a schlieren-apparatus

A schlieren-apparatus divides bent light from not sufficient bent light, the last is caught up by a schlieren-diaphragm, and imagery ensued only with bent light. Already Newton [4] III observation 5 had proved that bent light comes only out of the narrow surroundings of the slit (and not from the whole slit) what is not only confirmed by Young (he limited this falsely on the edges) but also by Fresnel [1] in his first paper in diffraction. It is therefore not astonishing but self-evident that in a schlieren-apparatus as images of edges appear only stripes in the surroundings of the images of edges, as already Nieke [5] reported.

In a schlieren-apparatus by Abbe (according Nieke [5] there fig. 1) a wire or a slit was placed in the same plane S, the photo-plate was placed in the plane S'. To obtain comparable results, wires of the same diameter formed the slit edges. For that purpose wires were glued to razor-blades and two in such way produced slit-edges formed the slit at which an equal wire served for adjusting the width of slit. At the diameter of 0.5 mm the wire consisted of axis-steal, at 0.2 mm a copper wire was stretched and glued at the razor-blade. At the diameter 0.1 mm a wire could not glued with necessary exactness at the edge of a razor-blade, hence the razor-blade was applied originally.
Fig. 3, 4, and 5 show photos in the schlieren-apparatus, in each case from the hindrance in a and b, and from the slit in c and d. The comparisation of the photos shows that within adjustment errors the images of wire and slit are in accordance. Hence it follows that in both cases bent light is coming out of

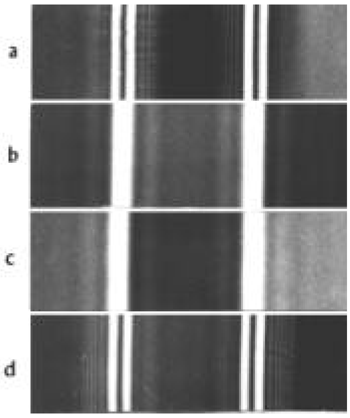

Figure 3. Images of hindrance and slit in a schlieren-apparatus after Abbe in Nieke [5] figure 1. Wire and slit 0.5 mm. a' : a - 4 and optical enlarged.

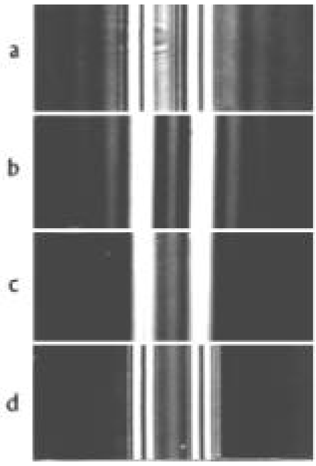

Figure 4. Image of hindrance and slit of the measurement 0.2 mm in a schlieren-apparatus. a ... d as figure 3.

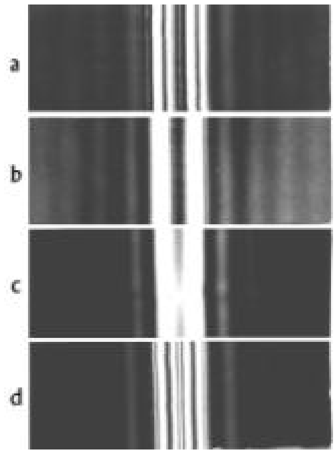

Figure 5. Image of hindrance and slit of the measurement 0.1 mm in a schlieren-apparatus. a ... d as figure 3.

a: wire image 2 mm before the cross-plane,
b: wire image in the cross-plane,
c: slit image in the cross-plane,
d: slit image 2 mm before the cross-plane.

analogous spheres. The images of edges in incident light accurately appear there where the dark stripe between double-stripe of every edge is laying. One part of every double-stripe seems to come from the jaws of hindrance or slit, therefore light must be displaced sideways as already reported by Nieke [5]. Here this appears particular important for only thereby the images of hindrance and slit in a schlieren-apparatus can be equal. At 0.1 mm measurement of hindrance or slit in fig. 5 touch both double-stripes.

Here is shown the by Nieke [5] only mentioned crossing-plane in the figures b and c, there the two part-stripes of one double-stripe cross according imagery of the second objective of schlieren-apparatus. Here are also to see additional diffraction fringes of schlieren-diaphragm which are on principle not avoidable. If the second objective of schlieren-apparatus has a large focus-length, the disturbance can be keep small. In large distances the intensities of minima turn lower. In every case the edges of schlieren-diaphragm shall be standing in a minimum of diffraction-figure.

An influence of radius of edges on diffraction-figures in the here used large distances was not found in accordance with literature. Only in short distances is partially reported about trifling influence.

<u>III. Diffraction at hindrance and slit without schlieren-apparatus</u>

A super-pressure mercury-lamp with green filter was imaged by a condenser on a 0.05 mm slit which is placed in the object-side focal-plane of a lens f’ = 320 mm. The diffraction object was placed direct behind this lens in the parallel optical path.

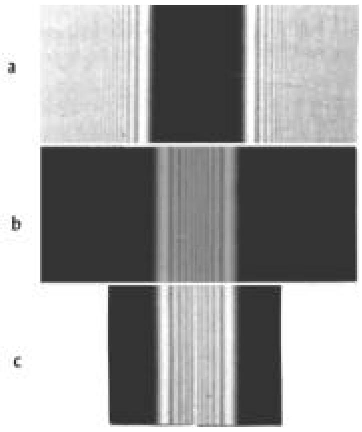

Figure 6. Diffraction-figures of hindrance and slit in 0.1 m distance without schlieren-apparatus in Fresnel’s manner of observation and in parallel incident light.
a: wire f 1.5 mm
b: slit 1.5 mm.
c: the photo a is cut in the middle to pieces and edge-symmetrical pasted.

Figure 7. Diffraction-figure of the same hindrance and slit as figure 6 but in 0.3 m distance. a ... c as in figure 6.

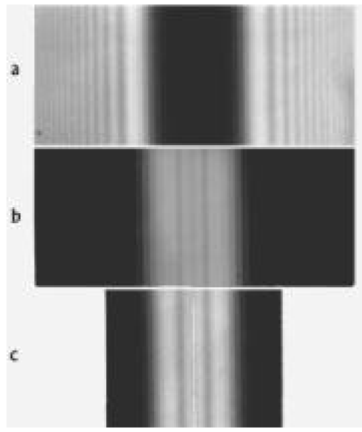

Fig. 6 shows the results in short distances which are yet so large that the in section 1 reported building-up of the diffraction-figure of half-plane was closed. The known diffraction-figure of the half-plane appears at the hindrance outside and at the slit inside, accordingly in every case light-sight of each edge. These fringes are designated as outer fringes of hindrance and inner fringes of slit (inside or outside of shadow-limits). In fig. 6 c the photo a is cut in the middle and pasted mirror-symmetrical to the edges.

Fig. 7 shows the diffraction-figures in something larger distances in the same arrangement. The fringes analogue the half-plane are still to find, they are broader and become fewer. In 7a the inner fringes of hindrance and in 7 b the outer fringes of slit are building up with constant intervals which are to find shadow-side in both cases, fig. 7 c is again cut and pasted edge-symmetrical.

In fig. 7 appear inner fringes of hindrance and outer fringes of slit which do not obey the fringes of half-plane and their distance-dependence. For these fringes grow linear to the distance, so an angle of diffraction can be attached. Their minima are to find in

$$\sin \alpha = n \lambda / d, \qquad \text{with } n = 1, 2, 3... \qquad (5)$$

and the maxima in

$$\sin \alpha = (2n + 1) \lambda / 2 d \qquad \text{with } n = 1, 2, 3..., \qquad (6)$$

and $\sin \alpha = 0$ for slit, wit h d as hindrance-breadth or slit-width and n the diffraction-order.

From the existence of outer fringes of hindrance and inner fringes of slit in short distances ensued, that the formulas (5) and (6) are not to extrapolate to the distance nought. This extrapolation is inadmissible. Out of the results of the schlieren-apparatus ensued that bent light comes only out of a narrow surroundings of the edges and not from the whole slit as formulas (5) and (6) after extrapolation presuppose. Consequently this extrapolation is also wrong, but this wrong and inadmissible extrapolation nevertheless is used till today by interpretation of diffraction with waves.

IV. Babinet's Theorem

In 1837 Babinet [6] found that hindrance and slit gave corresponding diffraction-figures. Hindrance and slit of same size are denoted as complementary screens.

Since about 1850 was used only Fresnel’s interpretation of diffraction with description of bounder-line cases in great distances, so did not be mentioned the inner fringes of slit in short distances. The characteristic of this appearance, the edge-symmetric exchanged phenomenon of diffraction at complementary screens, was left out and only special-cases are noticed. In the observation-manner of Fraunhofer [7], with imaging of light-source in the screen of diffraction-figure, appear only outer fringes of slit and inner fringes of hindrance. If the central-figure is took out so here agree the diffraction-figures of hindrance and slit. Only in this special-cases diffraction-figures are equal which in text-books are used e.g. in Bergmann a. Schaefer [8] and Pohl [9].

By Nieke [10] is detailed explained the motive for restriction on special-cases: The transition of inner to outer fringes of slit could not be explained by waves in Fresnel's interpretation. But with punctiform light-particles Newton could not explain diffraction. Therefore Newton's diffraction experiments, which described this transition in III observation 10, are simple let out and this transition was concealed. With fig. 6 and 7 is shown that first the consideration of inner and outer fringes gives here a real survey.

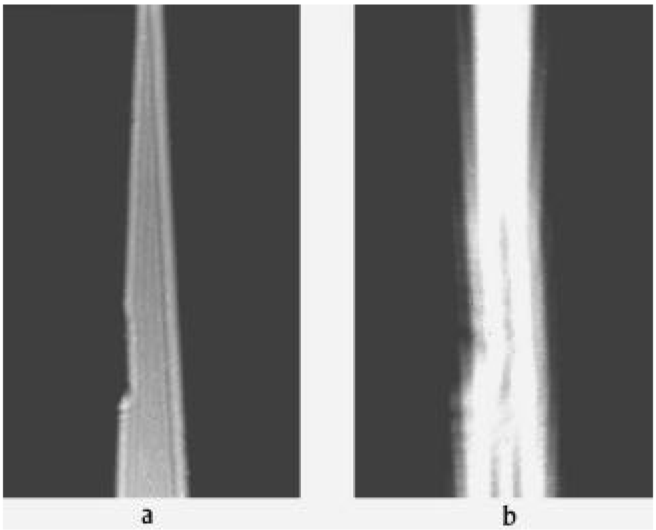

Figure 8. Diffraction-figures at a triangular-slit with a chip. A super-pressure mercury-lamp with a green filter was projected on a pinhole-diaphragm 0.05 mm. In 1 m distance stood a triangular-slit 0.. .3 mm with a copper-foil chip at one edge. That is in divergent light.
a: diffraction-figure in 0.1 mm distance,
b: diffraction-figure in 0.5 mm distance.

Laue [11] already directed that Babinet's theorem is not only valid for Fraunhofer's observation-manner. Maue [12] demanded for Babinet's theorem for complementary-screens complementary boundary-conditions. Complementary boundary-conditions are fulfilled by edge-symmetry because experiments in a schlieren-apparatus show that bent light comes from complementary spheres and diffraction-fringes are edge-symmetrical exchanged. Without the shadow-side displace of shadow-side bent light the agreement of diffraction at slit and hindrance was impossible, but only a edge-symmetric exchanged one.

V. Chips at the edges of slit and hindrance

Already Young and Fresnel fixed chips on the edges of slit and hindrance to explore their influence on the diffraction-figure. These experiments are examined and completed.

Fig. 8 a shows the diffraction at a triangular-slit with chip in short distances where appear only inner diffraction fringes. Accordingly in fig. 9 a at triangular-hindrance appear only outer fringes of hindrance. Both sort of fringes correspond to the diffraction of half-plane and it is to see that only are influenced the fringes on this side on which the chip is fixed. On this side the parallel fringes are distored but the intervals are preserved.

Different are the appearence in larger distances where in fig. 8 b also outer fringes of slit appear and in 9 b also inner fringes of hindrance. Here the chip effects on the whole diffraction-figure in height of chip as to see in fig. 8 b and 9 b. This effected a change of width of slit or breadth of hindrance and changed reciprocally the interval of fringes. Where simultaneous appear inner and outer fringes as in fig. 9 b there are to show both appearances in one figure.

VI. Inner and outer diffraction-fringes

General is known that intervals of diffraction-fringes of outer fringes of slit grow proportional to distance of slit. Therefore in formula (5) and (6) could ensure the statement of angles. At inner diffraction-fringes of slit and fringes of half-plane this is less known. Here is valid the formula (1) or by parallel incident light formula (2) where intervals grow with the root of distance. This entails that inner fringes of slit disappear in increasing distance in favour of outer fringes. So it is possible that also in Fresnel's observation-manner (without optics) appear diffraction-figures with only outer diffraction-fringes.

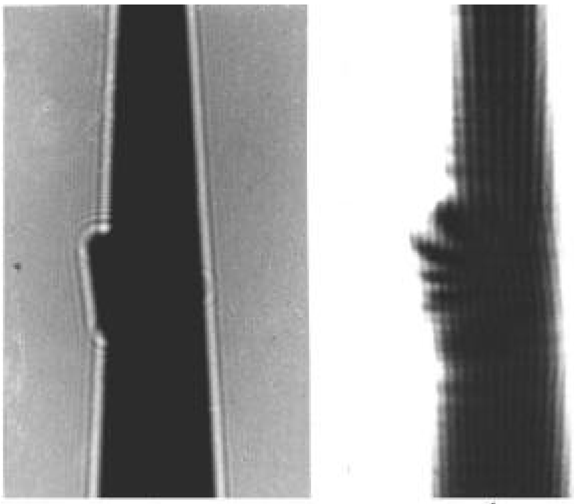
a b

Figure 9. Diffraction at a triangular-hindrance with a chip. Illuminated as described in figure 8 but in 1 m distance of the pinhole-diaphragm stood a triangular-hindrance 0 ... 3 mm with a chip on one edge.
a: diffraction-figures in 0.1 m distance,
b: diffraction-figures in 0.5 m distance.

To answer the question at which distance from the slit are to expect inner or outer fringes in Fresnel’s manner of observation. If in formula (2) for parallel incident light is set $x = d$ as shadowlimit, and $w = \sqrt{2}$ where therefore no minimum for inner fringes is existing, so results

$$s' = d^2 / \lambda. \qquad (7)$$

With X-rays Malgange a. Gronkowski [13] obtained the same result. For the transformation-sphere they established a computer-program for interpolation.

By inner fringes of slit the intervals of diffraction-fringes are dependent on the divergence of incident light according formula (1). On the contrary intervals of outer fringes of slit are scarcely not influenced by convergent or divergent illumination, but this illumination influenced very much intensity and distribution of light. So at all it is not easy to produce real intensity-moderate symmetric diffraction-figures.

VII. Discussion

As deficiency in description of diffraction at complementary screens is to find that consciously or unconsciously was avoided a discussion about inner and outer diffraction-fringes. That happened because the transition from inner to outer fringes of slit are not to explain hitherto.

Here Nieke [10] directed that the statement: „Suffice separated edges have to give the diffraction-figure of the half-plane“, is not valid but it is to explain why at the same slit-width in short distances results inner and in large distances outer fringes.

Only with Fraunhofer’s manner of observation this difficulty was removed for there appear (in the focus-plane) only outer diffraction-fringes. In a real description of Babinet's theorem eyes dare not shut against difficulty. In return for it yields a clear notion of Babinet's theorem if is accented the edge-symmetric exchange of diffraction-appearances. Then only in special-cases one finds agreement if the central-figure is took out.

Experiments with chips at edges show that one can experiment in the slit even if Born a. Biem [14] denied this, but it is to distinguish between inner and outer fringes. At inner fringes of slit and outer fringes of hindrance the chip effects only to the diffraction-figure of that edge on which it is fixed. In larger distances if appear outer fringes of the slit and inner fringes of hindrance then the chip effects on the whole diffractio n-figure of these fringes.

Lohmann a. Sinzinger [15] examined Babinet’s principle if only a part of a computer-holography was complementary exchanged. They termed this case as local Babinet-effect. The principle of edge-symmetric exchange can not be effective because they compare only with Fraunhofer's manner of observation.

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